1

Principles of Turbomachinery

1.1 TURBOMACHINE

While discussing the minimal number of components needed to constitute a heat engine [16], it was mentioned that mechanical energy output is obtained from an expander (work output device), whereas mechanical energy input to the system is due to a pump or a compressor which raises the pressure of the working fluid, a liquid or a gas. Both the expander and the pump (or compressor), are devices which provide work output or accept work input to affect a change in the stagnation state (Sec. 1.3) of a fluid. These devices are often encountered as parts of heat engines, though they can function independently as well. The principles of operation of both a work output device (e.g., an internal combustion engine of the reciprocating type) and a work input device (the reciprocating air-compressor [16]), have already been studied. In addition to these two types, there exist other devices which are invariably of the rotary[1] type where energy transfer is brought about by dynamic action, without an impervious boundary that prevents the free flow of a fluid at any time. Such devices are called *turbomachines*.

The turbomachine is used in several applications, the primary ones being electrical power generation, aircraft propulsion and vehicular propulsion for civilian and military use. The units used in power generation are steam, gas and hydraulic turbines, ranging in capacity from a few kilowatts to several hundred and even thousands of megawatts, depending on the application. Here, the turbomachine drives the alternator at the appropriate speed to produce power of the right frequency. In aircraft and heavy vehicular propulsion for military use, the primary driving element has been the gas turbine. The details of these types of machines will be provided in later chapters.

The turbomachine has been defined differently by different authors, though these definitions are similar and nearly equivalent. According to Daily [1], the turbomachine is a device in which energy exchange is accomplished by hydrodynamic forces arising between a moving fluid and the rotating and stationary elements of the machine. According to Wislicenus [2], a turbomachine is characterized by dynamic energy exchange between one or several rotating elements and a rapidly moving fluid.

[1] Rotary type machines such as gear pump and screw pump are positive displacement machines and work by moving a fluid trapped in a specified volume.

Binder [3] states that a turbomachine is characterized by dynamic action between a fluid and one or more rotating elements. A definition to include the spirit of all the preceding statements would be: *A turbomachine is a device in which energy transfer occurs between a flowing fluid and a rotating element due to dynamic action resulting in a change in pressure and momentum of the fluid*. Mechanical energy transfer occurs into or out of the turbomachine, usually in steady flow. Turbomachines include all those types that produce large-scale power and those that produce a *head* or pressure, such as centrifugal pumps and compressors.

The principal components of a turbomachine are: (*i*) A *rotating element carrying vanes* operating in a stream of fluid, (*ii*) A *stationary element* or elements which generally act as guide vanes or passages for the proper control of flow direction and the energy conversion process, (*iii*) an *input* and/or an *output* shaft, and (*iv*) a *housing* (Fig. 1.1). The rotating element carrying the vanes is also known by the names *rotor, runner, impeller,* etc., depending upon the particular application. Energy transfer occurs only due to the exchange of momentum between the flowing fluid and the rotating elements; there may not be even a specific boundary that the fluid is not permitted to cross. Details relating to these will be discussed in the following sections.

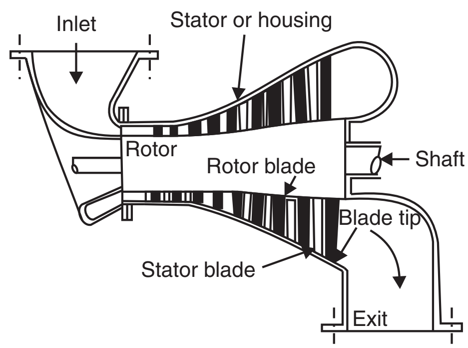

Fig. 1.1. Schematic cross-sectional view of a turbine showing the principal parts of the turbomachine.

The stationary element is also known by different names—among them *guide-blade* or *nozzle*—depending on the particular machine and the kind of flow occurring in it. A stationary element is not a necessary part of every turbomachine. The common ceiling fan used in many buildings to circulate air during summer and the table fan are examples of turbomachines with no stationary element. Such machines have only two elements of the four mentioned above: an input shaft and a rotating blade element.

Either an input or an output shaft or both may be necessary depending on the application. If the turbomachine is *power-absorbing*, the enthalpy of the fluid flowing through it increases due to mechanical energy input at the shaft. If the turbomachine is *power-generating,* mechanical energy output is obtained at the shaft due to a decrease in enthalpy of the flowing fluid. It is also possible to have power-transmitting turbomachines which simply transmit power from an input shaft to an output shaft, just like a clutch-plate gear drive in a car which transmits the power generated by the reciprocating engine to the shaft which drives the wheels. In principle, the device acts merely as an energy transmitter to change the speed and torque on the driven member as compared with the driver. There are many examples of a these types of machines. Examples of power-absorbing turbomachines are mixed-flow, axial-flow and centrifugal pumps, fans, blowers and exhausters, centrifugal and axial compressors, etc. Examples of power-generating devices are steam, gas and hydraulic turbines. The best known examples of power-transmitting turbomachines are fluid-couplings and torque-converters for power transmission used in automobiles, trucks and other industrial applications.

The housing too is not a necessary part of a turbomachine. When present, it is used to restrict the fluid flow to a given space and prevent its escape in directions other than those required for energy transfer and utilization. The housing plays no role in the energy conversion process. The turbomachine that has housing is said to be *enclosed* and that which has no housing is said to be *extended* [4]. The ceiling-fan shown in Fig. 1.2 is an example of an extended turbomachine and all the rest shown in the figure are *enclosed* turbomachines.

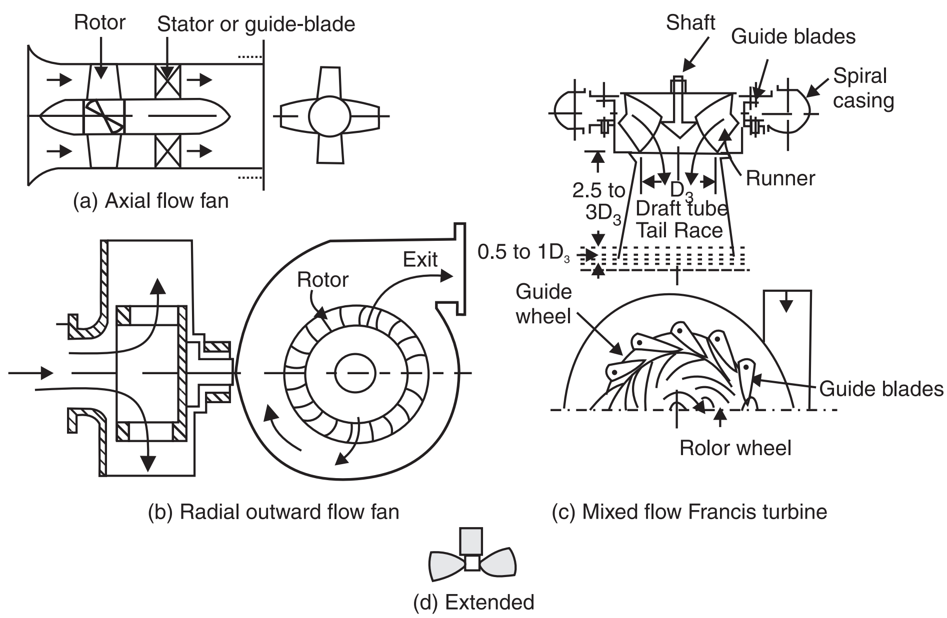

Fig. 1.2. Classification based on fluid flow in turbomachine.

Turbomachines are also categorized by the direction of fluid flow as shown in Fig. 1.2. The flow directions are: (*i*) axial, (*ii*) radial and (*iii*) mixed. In the axial-flow and radial-flow turbomachines, the major flow directions are approximately axial and radial respectively, while in the mixed-flow machine, the flow usually enters the rotor axially and leaves radially or vice versa. Mixed flow may also involve flow over the surface of a cone. An example of a mixed-flow machine is a mixed-flow pump. A radial flow machine may also be classified into *radial inward flow(centripetal) or radial outward flow(centrifugal) types* depending on whether the flow is directed towards or away from the shaft axis.

1.2 POSITIVE-DISPLACEMENT DEVICES AND TURBOMACHINES

In a positive-displacement machine[2], the interaction between the moving part and the fluid involves a change in volume and/or a translation of the fluid confined in a given boundary. During energy transfer, fluid expansion or compression may occur in a positive-displacement machine without an appreciable movement of the mass centre-of-gravity of the confined fluid. As such, changes in macroscopic kinetic energy and momentum may be neglected in most of these machines. The movement of a piston, screw or gear-tooth causes changes in fluid volume because of the displacement of the boundaries, i.e., the fluid cannot escape from the boundaries except due to unavoidable leakage. An expansion or contraction may occur if the fluid is compressible as for example, in a balloon being filled with air. The action is therefore nearly static and completely different from that of a turbomachine where the action is fast, dynamic and the energy transfer occurs without the necessity for a confining boundary. (In compressible flow handling machines, fluid flows at very high velocities close to acoustic speed at certain locations).

The differences between positive-displacement machines and turbomachines are clarified further by comparing their modes of action, operation, energy transfer, mechanical features etc., in the following:

- **Action:** A positive-displacement machine creates thermodynamic and mechanical action between a nearly static fluid and a relatively slowly moving surface. It involves a change in volume or a displacement (bodily movement of the confined fluid).

  A turbomachine creates thermodynamic and dynamic interaction between a flowing fluid and rotating element and involves energy transfer with pressure and momentum changes. There is no positive confinement of the fluid at any point in the system.

- **Operation:** The positive-displacement machine commonly involves reciprocating motion and unsteady flow of the fluid, though it is not impossible for the machine to have a purely rotary motion and nearly steady flow. Examples of such rotating positive-displacement machines are gear-pumps and screw-pumps. However, since the fluid containment is positive, stopping a positive-displacement machine during operation may trap a certain amount of fluid and maintain it indefinitely in a state different from that of the surroundings, if heat transfer and leakage are completely absent.

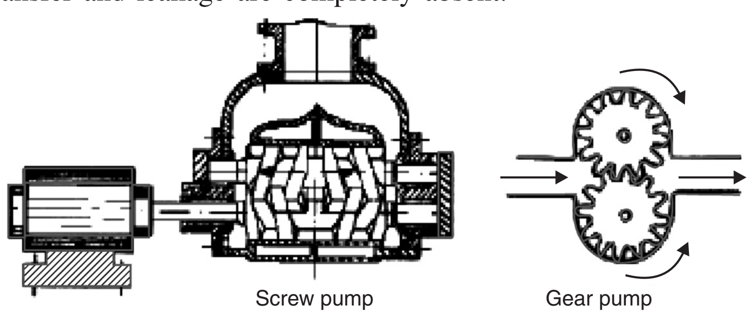

Fig. 1.3. Screw pump and gear pump.

[2] A positive-displacement machine is one which takes a fresh charge at the beginning of each cycle and discharges it at the completion of the cycle.

The turbomachine involves, in principle, steady flow of fluid and a purely rotary motion of the mechanical element. A turbomachine may also involve unsteady flow for short periods of time, especially while starting, stopping or during changes of loads. However, in most instances, the machine is designed for steady-flow operation. As there is no positive containment of the fluid, stopping of the machine will let the fluid undergo a change of state (in a matter of milliseconds), and become the same as that of the surroundings.

- **Mechanical Features:** The positive-displacement machine commonly involves rather low speeds and is relatively complex in mechanical design. It is usually heavy per unit of output and employs valves which are open only part of the time, as in reciprocating machines. Also, rather heavy foundations are usually needed because of reciprocating masses and consequent vibration problems. Generally in such machines, the mechanical features are more complex than in turbomachines.

  Turbomachines usually employ high rotational speeds, are simple in design principle and are generally light in weight per unit of power output. Their foundations may be quite light since vibration problems are not severe. They do not employ valves that open and close during steady-state operation. Usually, only inexpensive associated equipment is required.

- **Efficiency of Conversion Process:** In positive-displacement machines, the use of positive containment and a nearly static energy transfer process may result in a higher efficiency relative to that of a turbomachine which employs a dynamic process including high-speed fluid flow. The higher efficiency of energy conversion used to be the advantage of a positive-displacement machine as compared with a turbomachine which used to exhibit somewhat lower efficiencies. Compression by dynamic action often involves higher losses and hence lower efficiencies, though expansion in a turbomachine results in better efficiencies than in compression. Nevertheless, both these often used to fall short of the corresponding reciprocating machine in performance. In modern turbomachines which are designed through the aid of computers and high quality, efficient software, the difference between compressive and expansive efficiencies is not large. They are both high nearly 85–90%.

- **Volumetric Efficiency:** The volumetric efficiency of a machine with positive-displacement is normally well below that of turbomachines and in some cases, very low because of the opening and the closing of valves needed for continuous operation. In turbomachines, during steady state operation, there exist no inlet and outlet valves and the volumetric efficiency differs little from 100%. Also, since the flow is continuous and the fluid velocities are high, a turbomachine has a high fluid handling capacity per kilogram weight of the machine. As an example, a 300 kW gas turbine plant typically handles about 22 $kg \cdot s^{-1}$ of air and has a weight of 900 kg. Thus, the specific power output of this plant is 15 $kJ \cdot kg^{-1}$ of air and it has a power plant weight per unit mass flow rate of air between 10 and 100. In comparison, an aircraft reciprocating power plant producing 300 kW handles 2 $kg \cdot s^{-1}$ of air and has a weight of 1000 kg. Thus, its specific weight is 150 per $kg \cdot s^{-1}$ of air flow. For all types of industrial power plants, the specific weight of a reciprocating plant is about 10–15 times that of a turbo-power plant.

- **Fluid Phase Change and Surging:** Phase changes occurring during flow through a turbomachine can frequently cause serious difficulties to smooth operation. Examples of these are cavitation at pump inlets and hydraulic turbine outlets as well as condensation in steam turbines resulting in blade erosion and/or a deterioration of machine performance. Surging or pulsation (Chapter 6, a phenomenon associated with turbomachinery), is caused by an unstable flow situation due to a rising head-discharge characteristic. It is characterized by the pulsation of fluid pressure between the inlet and the outlet of the turbomachine, i.e., the reversal of flow direction accompanied by violent flow fluctuations. The machine may vibrate violently, and under certain operating conditions, may even be damaged by these vibrations. The performance of the device deteriorates considerably even when the flow fluctuations are not violent. Problems of phase change pulsation and surging are of no importance in positive-displacement machines.

1.3 STATIC AND STAGNATION STATES

In dealing with turbomachines, one is concerned with fluids, often compressible and moving at high speeds exceeding the speed of sound. Even in turbomachines dealing with incompressible fluids where the velocities are relatively low, the kinetic and potential energies of the fluid are often large and constitute major fractions of the total energy available for conversion into work. Simplistic approaches which neglect potential and kinetic energies cannot provide sufficiently accurate results for design. It is therefore necessary to formulate equations based on the actual state of the fluid including all the energies at the given point in the flow. Taking these factors into account, we use the equations of the First and Second laws of Thermodynamics to specify two fluid states called respectively, the 'Static' and the 'Stagnation States' which will be discussed below.

- **The Static State:** First, consider a fluid flowing at a high speed through a duct. In order to measure the properties of the fluid, one may insert an instrument such as a pressure gauge or a thermometer at some point in the flow. One can imagine two types of measurements, one in which the measuring instrument moves at the *same local speed* as that of the fluid particle and another in which it is *stationary with respect to the particle* the properties of which are under investigation. Measurements of the first type made with an instrument which moves *with the same local speed as the particle* are said to determine a 'static' property of the fluid. Note that what is stationary is neither the fluid nor the instrument to measure the property—both of them may move except that the measuring instrument moves at the same speed as the fluid locally and is therefore *at rest with respect to the particle of the fluid*. For example, one can consider a pressure measurement made with a static pressure gauge which is usually fixed to the side of the duct. In this case, the fluid particle and the instrument are *at rest with respect to each other at the point where the measurement is being made*. Hence, the measured pressure is a static pressure. Any measurement made in consonance with this stipulation determines a static property, be it one of pressure, temperature, volume, or any other, as specified. The state of the particle fixed by a set of static properties is called the '*Static State'*.

- **The Stagnation or Total State[3]:** The *stagnation state is defined as the terminal state of a fictitious, isentropic, work-free and steady-flow process during which the macroscopic kinetic and potential energies of the fluid particle are reduced to zero*, the initial state for the process being the static state. The macroscopic kinetic and potential energies are those measured with respect to an arbitrary and pre-specified datum state.

The stagnation state as specified above is not representative of any true state of the fluid. No real process leads to the stagnation state, because no real process is truly isentropic and perfectly free from thermal exchange with the surroundings. Despite the impossibility of achieving it, if proper care is taken to account for errors in measurement and appropriate corrections incorporated, many of the properties measured with instruments like Pitot tubes, thermocouples, etc., do provide readings that approximate stagnation properties closely. Further, stagnation property changes provide ideal values against which real machine performance can be compared. These properties and the state defined by them (the stagnation state), are thus of great importance in turbomachinery.

By using the definition of a stagnation state, it is possible to obtain expressions for stagnation properties in terms of static properties. Considering any steady-flow process, the First Law of Thermodynamics [15] gives the equation:

$$q - w = \Delta h + \Delta ke + \Delta pe \qquad \ldots(1.1)$$

where, q and w are respectively the energy transfers as heat and work per unit mass flow, h is the static enthalpy and ke and pe are respectively the macroscopic kinetic and potential energies per unit mass. It is known that $ke = V^2/2$, and $pe = gz$, V, being the fluid particle velocity and z, the height of the particle above the datum at the point under consideration.

Since the static state is the initial state in a fictitious isentropic, work-free, steady flow process and the stagnation state is the terminal state where both the kinetic and potential energies are zero, the difference in enthalpies between the stagnation and static states is obtained by setting $q = w = 0$, $\Delta h = h_o - h_i$, $ke_o = 0$ and $pe_o = 0$ in Eq. (1.1). There is then obtained:

$$h_o - (h_i + ke_i + pe_i) = 0 \text{ or, } h_o = (h + ke + pe) \qquad \ldots(1.2)$$

where, the subscript o represents the stagnation state and i represents the initial static state. Equation (1.2) follows from the fact that at the stagnation state, both the kinetic and the potential energies are zero. In the last part of Eq. (1.2), the subscript, i, has been removed and from here onwards, the properties at the static state will be indicated without the subscript as shown. The enthalpy h_o in the stagnation state has thus been expressed in terms of three known properties, h, ke and pe of the static state.

As stated earlier, it is necessary that the process changing the state from static to stagnation be isentropic, i.e., $s_o = s$, and hence, the entropy in the stagnation state is equal to the entropy in the static state. Thus, two independent stagnation properties, namely the enthalpy h_o and the entropy s_o, have been determined in terms of the known properties at the static state. Since

[3] This definition was given by Dean R.H. Zimmerman who was Visiting Professor of Mechanical Engineering at the start of IIT-K, 1962–1967.

according to the 'State Postulate', the knowledge of any two independent properties at a specified state is sufficient to fix the state of a simple compressible substance [15], the stagnation state is totally determined and it should be possible to determine any other required property of the stagnation state in terms of the two known properties, h_o and s_o. Also, according to the Second Law of Thermodynamics, since $T \cdot ds = dh - vdp$, and the entropy remains constant in the change from static to stagnation state, $ds = 0$ and $dh = vdp$, $v = 1/\rho$, being the specific volume and ρ, the density. Hence, integration yields for the change from static to stagnation state:

$$h_o - h = \int dh = \int vdp \qquad \ldots(1.3)$$

The integration on the right hand side depends on the variation of volume with respect to pressure in an isentropic process. If the p-v property relation is known, the equation above may be integrated and one can determine the stagnation pressure p_o in terms of the static enthalpy h and the static pressure p. This will be done for two special cases.

(***a***) **Incompressible Fluid:** For an *incompressible fluid*, $dv = d(1/\rho) = 0$, ρ being the density of the fluid. Since the density is constant and independent of state:

$$h_o - h = (p_o - p)/\rho$$

Thus, on using Eq. (1.2), $$p_o/\rho = p/\rho + (h_o - h) = p/\rho + V^2/2 + gz \qquad \ldots(1.4)$$

The stagnation pressure of the incompressible fluid has now been expressed in terms of its static pressure, velocity and height above a specified datum. According to the First Law of Thermodynamics, *the stagnation enthalpy* h_o *and the stagnation pressure* p_o *should be constant along any streamline which experiences no energy transfer as heat or as work*. Hence, for an incompressible, frictionless fluid in steady flow, it is seen that the stagnation pressure remains a constant along a streamline in an un-accelerated coordinate system. This is the Bernoulli's theorem studied in Fluid Mechanics.

In addition, for a change from the static to the stagnation state of an incompressible fluid, since there is no entropy change and $pdv = 0$, $Tds = du + pdv$ yields $du = 0$. Hence, $u = u_o$, i.e., the *internal energies of an incompressible fluid in the static and stagnation states are equal*. Moreover, since the internal energy of an incompressible fluid is a function of temperature alone, one concludes that:

$$u_o - u = c(T_o - T) = 0, \text{ i.e., } T_o = T \qquad \ldots(1.5)$$

The local static and stagnation temperatures are equal to each other at every point in incompressible and loss-free fluid flow.

(***b***) **Perfect Gas:** Since the enthalpy of a perfect gas is a function of temperature alone, from Eq. (1.2), with $ke = V^2/2$ and $pe = gz$ one gets:

$$c_pT_o = c_pT + V^2/2 + gz \text{ or } T_o = T + (V^2/2 + gz)/c_p \qquad \ldots(1.6)$$

In compressible flow machines, fluid velocities vary from about 60 m·s^{-1} to 600 m·s^{-1} or more, whereas the maximum value of z is rarely in excess of 4 m in most steam and gas turbines. As such, even at the minimum flow velocity,

$$V^2/2 = 60^2/2 = 1800 \text{ J·kg}^{-1} \text{ and, } gz = (9.81)(4) = 39.24 \text{ J·kg}^{-1}.$$

The calculations above indicate that the magnitude of kinetic energy is far in excess of the potential energy in most compressible flow machines. It is therefore usual to neglect the term gz in comparison with the term $V^2/2$ and to write the equation to compute the stagnation temperature of a perfect gas in the form:

$$T_o = T + V^2/(2c_p) \qquad \ldots(1.7)$$

One can now determine the stagnation temperature by using the substitution $c_p = \gamma\mathfrak{R}/(\gamma - 1)$, to obtain:

$$T_o = T[1 + (\gamma - 1)V^2/(2\gamma\mathfrak{R}T)] \text{ or} \qquad \ldots(1.8a)$$

$$p_o v_o = pv[1 + (\gamma - 1)M^2/2] \qquad \ldots(1.8b)$$

where M, is the local Mach number of a perfect gas defined by the equation $M = V/a$, in which the speed of sound in the gas at the static temperature T, is denoted by the symbol $a = (\gamma\mathfrak{R}T)^{1/2}$. Again, since for the isentropic expansion of a perfect gas, $v_o/v = (p/p_o)^{1/\gamma}$, one can write:

$$(p_o/p) = \beta^{\gamma/(\gamma - 1)}, \qquad \beta = 1 + (\gamma - 1)M^2/2 \qquad \ldots(1.9a)$$

With this simplification in notation, the expressions for the stagnation temperature T_o, (Eq. 1.8a) and stagnation pressure p_o (Eq. 1.9a) may be rewritten in the forms:

$$T_o = T\beta, \text{ and } p_o = p\beta^{\gamma/(\gamma - 1)} \qquad \ldots(1.9b)$$

Example 1.1. Dry saturated steam at 1 atm. static pressure flows through a pipe with a velocity of 300 $m{\cdot}s^{-1}$. Evaluate the stagnation (total) pressure and the stagnation temperature of the steam: (a) By using steam tables and (b) by assuming steam to behave as a perfect gas with $\gamma = 1.3$.

Data: Saturated steam flow, static pressure $p = 1.013$ bar, velocity $V = 300$ $m{\cdot}s^{-1}$ (bar = 10^5 Pa)

Find: The stagnation pressure p_o and the stagnation temperature T_o, (*i*) Use steam tables (*ii*) Treat steam as a perfect gas with $\gamma = 1.3$.

Solution: In working this example and other examples in Chapters 1 and 2, the use of Steam Tables and Mollier chart will be exhibited, though it is possible to solve the problem with the help of a computer program without the use of either the tables or the chart. The procedure for writing a computer program to solve similar problems will be provided in Chapter 3.

(*i*) The static temperature corresponding to a saturation pressure of 1.013 bar is $T = 100$°C. By referring to the steam tables for the properties of saturated steam at the temperature 100°C (Table A.2 from Appendix A), we get for the static enthalpy, $h = 2675.9$ $kJ{\cdot}kg^{-1}$ and for the saturation static entropy $s = 7.3549$ $kJ{\cdot}kg^{-1}K^{-1}$. Hence, from Eq. (1.2), neglecting potential energy (since steam is a compressible substance and its potential energy is small), one gets for the stagnation enthalpy,

$$h_o = h + ke + pe = 2675.9 + 300^2/(2{\times}1000) = 2720.9 \text{ kJ}{\cdot}\text{kg}^{-1}.$$

Stagnation entropy, $s_o = s$ (static entropy) = 7.3549 $kJ{\cdot}kg^{-1}$ K^{-1}.

On referring to the Mollier chart with the values of h_o and s_o specified above, we get:

Total pressure, $\boldsymbol{p_o}$ **= 1.246 bar;** Total temperature $\boldsymbol{T_o}$ **= 121°C.**

(*ii*) If steam behaves as a perfect gas with $\gamma = 1.3$ and $\mathfrak{R} = 8317/18 = 462.06$ J·kg^{-1} K^{-1},

$$\boldsymbol{T_o = T[1 + (\gamma - 1)V^2/(2\gamma\mathfrak{R}T)]}$$

$$= 373.15\{1 + 0.3\times300^2/[2(1.3)(462.06)(373.15)]\} = 395.63 \text{ K} = \mathbf{122.5\,°C}$$

This newly calculated value of total temperature agrees reasonably well with that calculated by using Mollier chart and is therefore satisfactory. Then, stagnation pressure:

$$\boldsymbol{p_o = p(T_o/T)^{\gamma/(\gamma-1)}} = 1.013(395.63/373.15)^{3.5} = \mathbf{1.243\ bar}$$

The newly computed pressure agrees even better with the previous value obtained by using steam tables. This is the reason that for quick calculations, we can simply use the perfect gas equations and obtain reasonably good results. The fact that superheated steam behaves nearly like a perfect gas can be used to obtain quick approximations to the properties when the pressure is well below critical. This fact will be utilized in Chapter 3 to write a computer program to calculate the states of superheated steam undergoing an isentropic expansion.

1.4 FIRST AND SECOND LAWS OF THERMODYNAMICS APPLIED TO TURBOMACHINES

Fluid flow in turbomachines always varies in time, though it is assumed to be steady when a constant rate of power generation occurs on an average. The variations are due to small load fluctuations, unsteady flow at blade-tips, the entry and the exit, separation in some regions of flow etc., which cannot avoided, no matter how good the machine and load stabilization may be. Similar statements can be made for power absorbing turbomachines as well. Nevertheless, on an overall basis when the average over a sufficiently long time is considered, turbomachine flows may be considered as steady. This assumption permits the analysis of energy and mass transfer by using the steady-state control volume equations. Assuming further that there is a single inlet and a single outlet for the turbomachine across the sections of which the velocities, pressures, temperatures and other relevant properties are uniform, one writes the steady flow equation of the First Law of Thermodynamics in the form:

$$\dot{Q} + \dot{m}\,(h_1 + V_1^2/2 + gz_1) = P + \dot{m}\,(h_2 + V_2^2/2 + gz_2) \qquad \ldots(1.10)$$

Here, $\dot{Q}$ = Rate of energy transfer as heat across the boundary of the control volume,

P = Power output due to the turbomachine, and

$\dot{m}$ = Mass flow rate.

Note: While making calculations, if enthalpy h is expressed in kJ·kg^{-1}, then both the kinetic and potential energy terms, $V^2/2$ and gz in Eq. (1.10) and other similar equations should be divided by 1000 during calculations.

Since $h_o = h + V^2/2 + gz$, (Eq. 1.2), one obtains:

$$q - w = \Delta h_o,$$

where, $\Delta h_o = h_{o2} - h_{o1}$, …(1.11a)